Palo Alto Girls Math Tournament 2026, Junior Division

Full name: _____

School: _____

Do not begin the test until you are instructed to do so. Read all rules carefully.

- **Basic Information:**

  The test: You have 31 questions total.

  Time: You have 1 hour to finish.

  What to use: Use only a pencil, pen, and eraser.

  What NOT to use: No calculators, rulers, or notes are allowed.

- **Answer Format:**

  Questions 1 to 30: Every answer must be a positive whole number (like 5, 24, 100). No fractions, no decimals, and no negative numbers.

  Question 31: This is an estimation question. Put an answer that you believe is closest to the exact answer. Make sure that your answer is a positive whole number (so same as above) with at most 10 digits.

- **Grading and Tiebreaking:**

  Scoring: You get 1 point for every correct answer on the first 30 questions.

  Guessing: You do NOT lose points for wrong answers. If you don't know the answer, take a guess!

  Tiebreaker: Question 31 is very important. If you and another student have the same score, the person with the closest guess on Question 31 wins. Make sure to put an answer for Question 31. If you don't, you will be ranked below all the other people that got the same number of points as you on Questions 1-30.

- **Important Reminders:**

  Diagrams: If you see a diagram or a shape, it may not be perfect (not drawn to scale).

  Questions: The proctors cannot help you solve the math problems or explain them to you.

  Remember: This test is not meant to be easy! Don't worry if you aren't able to solve many problems. It's not just you. If it's hard for you, it's hard for everyone else too! Also, remember that the questions are only roughly in difficulty order. If you get stuck for a long time, it is highly recommended to skip it, because you may find an easier question later on.

Good luck and have fun!

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Testsolvers: Raymond Zhou, Zhendi Cao, Jiaming Zhao

1. _____ What is $4 + 7 - 3 + 8 - 12 + 6 - 23 + 58$?

2. _____ What is $321 - 123 + 231 - 213$?

3. _____ What is the sum of the integers from 1 to 19, inclusive (including both 1 and 19)?

4. _____ How many digits does 10^{11} have?

5. _____ What is the sum of the first 6 prime numbers?

6. _____ What is the measure of an interior angle of a regular octagon in degrees?

7. _____ What is the value of $(1 \div 20) \div (3 \div 40) \div (5 \div 60)$?

8. _____ If $\sqrt{\frac{144}{x}} = 2$, find x.

9. _____ Pauline's Pizzeria offers three pizza styles, cheese, pepperoni and Hawaiian, available on either a thick or thin crust. Customers also must select exactly one topping from a list of olives, onions, mushrooms and anchovies. How many possible different pizzas are there?

10. _____ What is the maximum number of intersections between a rectangle and a circle?

11. _____ A cat is currently 2 years younger than a dog. Four years ago, the dog was twice as old as the cat. What is the sum of the cat's and the dog's current ages, in years?

12. _____ You have a 5 by 5 grid where the rows and columns are each numbered from 1 to 5. How many cells are there such that the row number plus the column number is divisible by 3?

13. _____ Let the probability of obtaining a sum of 6 when rolling 2 fair six-sided dice be $\frac{m}{n}$, where $\gcd(m, n) = 1$. What is $m + n$?

14. _____ How many sides does a regular polygon with an external angle of 3° have?

15. _____ Define a function $b(x) = x + 4$ and a function $r(x) = \frac{x}{2}$. What is the value of $b(b(r(b(r(r(1024))))))$?

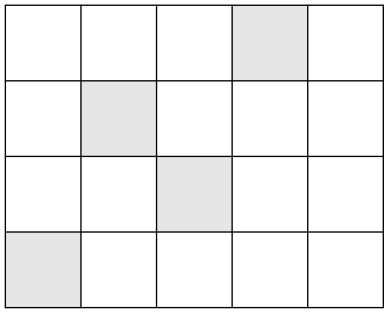
16. _____ Pal has 1 red pencil, 2 blue pencils, and 3 green pencils. If pencils of the same color are indistinguishable and the order of the pencils in the combination does not matter, how many combinations of 3 pencils can Pal pick?

17. _____ Marvin the mouse is trying to get from one end of a field to the other. The field is 120 meters long. Marvin can run 15 meters in 8 minutes, but then he will become tired and need a 3 minute rest. How many minutes will it take Marvin to reach the other side of the field?

18. _____ What is the hypotenuse of a triangle with sides in the ratio of $3 : 4 : 5$ and area 54?

19. _____ How many positive integers less than 100 are divisible by 2 and 3 but not 4?

20. _____ In the plane figure shown below, 4 of the unit squares have been shaded. What is the least number of additional unit squares that must be shaded so that the resulting figure has two lines of symmetry?



21. _____ I choose a random point inside my equilateral triangle ABC. What is the reciprocal of the probability that it is closest to A, second closest to B, and farthest from C?

22. _____ A cat is currently sleeping on a couch. Every 6 seconds, this cat purrs. A dog is currently chasing a squirrel. Every 5 seconds, this dog barks. A Paly student is preparing for their math test. Every 21 seconds, this student clicks their pen. At noon, the cat purrs, the dog barks and the student clicks their pen, all at the same time. How many seconds will pass before this happens again?

23. _____ Let a, b, c and d be integers such that $2^a 3^b 5^c 7^d = 882$. What is the value of $2a + 3b + 5c + 7d$?

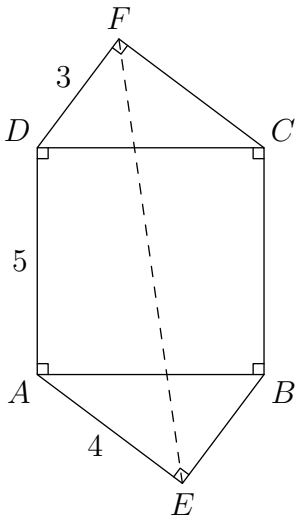
24. _____ Andrew has several blocks, each of which has a letter on it. He takes out 8 blocks and orders them in a line so that they spell out "PALOALTO". He then rearranges the letters into some other sequence. How many new sequences could Andrew have rearranged the blocks into?

25. _____ How many positive integers less than 10,000 have an even number of digits?

26. _____ Find the value of x if $(x-50)^2-(x-53)^2=2025$.

27. _____ 16 students are standing in a circle, playing a game of catch. Each second, a student passes the ball to either the person directly opposite to them, to the person to their right, or to the person to their left. Sally starts with the ball. After 4 tosses, how many students could possibly have the ball?

28. _____ In the diagram below, $ABCD$ is a square and $\triangle ABE$ and $\triangle CDF$ are right triangles. If $ABCD$ has a side length of 5, $AE = 4$ and $DF = 3$, what is $(EF)^2$?



29. _____ What is the sum of all 2-digit whole numbers that have exactly 8 proper divisors? (We say that x is a proper divisor of y if x and y are positive integers where x is less than y and $\frac{y}{x}$ is a whole number.)

30. _____ Let H be a regular hexagon. Let H' be the new hexagon formed by the midpoints of the sides of H. What is the sum of the numerator and denominator of the simplified ratio between the areas of H' to H?

31. _____ **ESTIMATION.** How many digits does 200! have? Recall that $n!$ is the *factorial* of n, and it is equal to $n \times (n-1) \times (n-2) \cdots \times 2 \times 1$. For example, $4! = 4 \times 3 \times 2 \times 1 = 24$.

Remember that leaving this question blank means that you will be ranked the lowest among all students who tied with you on the math portion of the test.

PAGMT 2026, Junior Division, Solution Guidelines

Sayan Singh

1 Preface

For PAGMT, we provide solution guidelines rather than full solutions because we want students to experience the process of discovery for themselves. Mathematics is not just about reaching the correct answer, but about developing creative approaches, testing ideas, and learning how to reason independently. These guidelines are meant to offer direction when needed while still leaving room for students to explore, make connections, and build confidence in their own problem-solving abilities. We have also attached an answer key for students to compare their final answers with. We hope that everyone can gain insight and knowledge into creative problem solving and mathematics with the help of this document.

2 Solution Guidelines

1. Combine the positive terms and negative terms separately, then subtract.

2. Group the numbers in a convenient way, such as pairing similar three-digit numbers.

3. Use the formula for the sum of consecutive integers, or pair terms from the beginning and end.

4. Recall that 10^n is a 1 followed by n zeros, so there are $n + 1$ digits.

5. List the first six prime numbers carefully, then add them.

6. Use the formula for the interior angle of a regular polygon, or divide the octagon into triangles.

7. Remember PEMDAS. Rewrite each division as multiplication by the reciprocal, then simplify carefully.

8. Interpret the equation carefully. Undo the square root operation by squaring both sides to solve for the missing value.

9. Use the multiplication principle of counting: choose one style, one crust type, and one topping.

10. Draw a square. Then, draw a slightly bigger circle than what fits inside it completely.

11. Let the cat's current age be a variable, then express the dog's age in terms of it. Translate the "four years ago" condition into an equation.

12. List the sum of the row number and column number in each box, and then count the boxes that have a number inside them divisible by 3.

13. Count the number of outcomes that give the required sum out of the total possible dice outcomes. Then simplify the fraction.

14. For a regular polygon, all exterior angles add to 360 degrees. Divide 360 by the given exterior angle to get the number of sides.

15. Work from the inside outward. Apply each function one step at a time, simplifying whenever possible. Make sure your calculations are correct!

16. List possible triples by number of red, blue, and green pencils, making sure not to exceed the available quantity of each color.

17. Determine how many running intervals are needed to cover the full distance. The most common incorrect answer here was 88. Remember that Marvin does not need to rest after finishing. (So he runs, rests, runs, rests, ..., rests, runs. He does not rest at the end, as he is finished.)

18. Use the side ratio to write the sides as multiples of a common scale factor. Use the area formula to find the scale factor.

19. "Divisible by 2 and 3" means divisible by 6. Count the number of multiples of 6 that are less than 100, and then remove those also divisible by 4.

20. Test possible pairs of symmetry lines. Reflect the already shaded squares across those lines and count the number of new squares needed.

21. Because the equilateral triangle is so symmetric, each ordering of A, B, C in closeness to point P is equally likely.

22. Find when all three events occur together again by computing the least common multiple of 6, 5, and 21.

23. Factor the given number into powers of 2, 3, 5, and 7. Match the exponents to the variables. Then evaluate the requested expression.

24. Count the distinct arrangements of the letters in "PALOALTO," accounting for repeated letters. Then subtract the original arrangement if the question asks for new sequences. The most common incorrect answer here was 5040, as many students forgot to subtract the original arrangement. (The problem specifically states "some other sequence" and "new sequences" explicitly.)

25. Count positive integers with 2 digits and 4 digits, since both are less than 10,000.

26. Use difference of squares to try to transform the equation into a simpler one.

27. Draw and label the 16 students by position. After each toss, track which positions are reachable using moves of left, right, or directly opposite. Make sure to consider all possibilities.

28. There are many ways to solve this problem, most involving the Pythagorean Theorem and finding the heights of triangles. Try to make as many new right triangles as you can, and find all the lengths.

29. A number with exactly 8 proper divisors has exactly 9 total positive divisors. Look for two-digit numbers whose prime factorizations give 9 divisors.

30. Draw or imagine the smaller hexagon formed by side midpoints. Use similar geometry, side lengths, or area decomposition to compare the area of the inner hexagon to the original.

31. **ESTIMATION.** Estimate the number of digits by approximating $\log_{10}(200!)$. Recall that $\log ab = \log a + \log b$. Use this property extensively to obtain the result.

3 Answer Key

1. 45
2. 216
3. 190
4. 12
5. 41
6. 135
7. 8
8. 36
9. 24
10. 8
11. 14
12. 9
13. 41
14. 120
15. 138
16. 6
17. 85
18. 15
19. 8
20. 10
21. 6
22. 210
23. 22
24. 5039
25. 9090
26. 389
27. 9
28. 98
29. 36
30. 7
31. 375