Palo Alto Girls Math Tournament 2026, Senior Division

Full name: ____

School: ____

Do not begin the test until you are instructed to do so.
Read all rules carefully.

- *Test Format*: This competition consists of 30 math problems and 1 estimation problem to be solved in 1 hour.
- *Permitted materials*: pencils, pens, erasers, scratch paper. Scratch paper will be provided by the proctors. Calculators, books, notes, rulers, compasses, protractors, or any other aids are prohibited.
- *Answer format*: Answers must be legibly written on the answer blank to be graded. Units are not required. For problems 1-30, all answers will be positive integers. For the estimation question, you may submit a positive integer that has a maximum of 10 digits.
- *Diagrams*: Diagrams are not necessarily to scale.
- *Scoring*: Your score will be the number of correct answers on questions 1-30. There is no penalty for guessing. Problem 31, the estimation question, will only be used for breaking ties. In the event of a tie, the student with a closer estimate to the actual answer will be ranked higher. If you leave the estimation question blank, you will be ranked last among all students who tied with you, so **it is highly recommended to answer the estimation question, even if you only spend a couple of seconds on it.**
- *Ties*: What if there is still a tie? In the unlikely case where two students tied and their estimations are equal, the person who solved the harder set of math questions will be ranked higher (the difficulty of the set of solved problems will be calculated by summing the problem numbers).
- *Additional*: Proctors may not answer any questions about the test problems.

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Testsolvers: Raymond Zhou, Zhendi Cao, Jiaming Zhao

1. ______________ What is the value of $(1 \div 20) \div (3 \div 40) \div (5 \div 60)$?

2. ______________ Aya wants to buy a pair of jeans for \$50 dollars. She first applies a 16% off coupon, and then uses a \$10 gift card. How many dollars does she have left to pay?

3. ______________ You have a set of 3 distinct integers. If the minimum is 1, the median is 2, and the average is 3, what is the maximum?

4. ______________ You have a 5 by 5 grid where the rows and columns are each numbered from 1 to 5. How many cells are there such that the row number plus the column number is divisible by 3?

5. ______________ Pal has 1 red pencil, 2 blue pencils, and 3 green pencils. How many combinations of 3 pencils can Pal pick, if pencils of the same color are identical and the order of the pencils in the combination does not matter?

6. ______________ A dodecahedron is a three-dimensional solid with 12 faces. Sharon takes one dodecahedron and applies glue to one of its faces. Sharon then takes another identical dodecahedron and applies glue to one of its faces. If Sharon connects the two dodecahedra so the faces with glue on them perfectly coincide, how many faces does the new solid have?

7. ______________ What is the hypotenuse of a triangle with sides in the ratio of $3:4:5$ and area 54?

8. ______________ Define a function $b(x) = x + 4$ and a function $r(x) = \frac{x}{2}$. What is the value of $b(b(r(b(r(r(1024))))))$?

9. ______________ Amy draws a circle and square so that the circumference of the circle is equal to the perimeter of the square. If the square has area π^2, what is the radius of the circle?

10. ________________ Let p be the probability of obtaining a sum of 6 when rolling 2 fair six-sided dice. What is the sum of the numerator and denominator of p, when p is expressed as a fraction in simplest form?

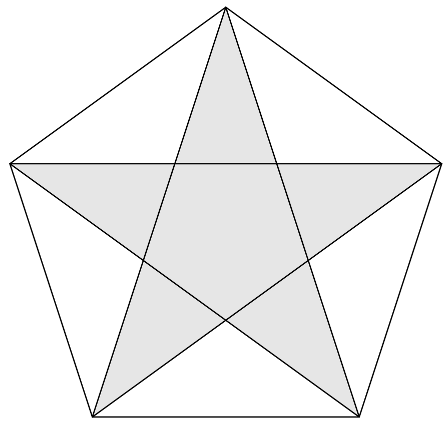
11. ________________ Let a, b, c, and d be integers such that $2^a 3^b 5^c 7^d = 882$. What is the value of $2a + 3b + 5c + 7d$?

12. ________________ Marvin the mouse is trying to get from one end of a field to the other. The field is 120 meters long. Marvin can run 15 meters in 8 minutes, but then he will become tired and need a 3 minute rest, before he is able to run for 8 minutes again, and so on. How many minutes will it take Marvin to reach the other side of the field?

13. ________________ Let K be the value $1! + 2! + \cdots + 2026!$ What is the units digit of K? Note that $n! = n \cdot (n-1) \cdot (n-2) \dots (2) \cdot (1)$.

14. ________________ How many digits does $3^2 \cdot 4^{12} \cdot 5^{22}$ have?

15. ____________ We can construct a five-pointed star by taking a regular pentagon and drawing its diagonals, and then shading in some of the area as shown below. What is the measure of the acute interior angle in such a star in degrees?



16. ____________ Find the sum of all positive integer divisors of 120.

17. ____________ Starting with 2 empty jugs with capacities of 3 liters and 10 liters, the goal is to have one jug holding 5 liters. There are 3 moves that you can perform:

- **Fill:** Fill either jug to its maximum capacity via an external water source.
- **Empty:** Completely empty the contents of either jug into a drain.
- **Transfer:** Pour water from one jug to the other. This action stops when either one jug becomes empty, or the other jug becomes full.

What is the minimum number of moves needed to reach your goal?

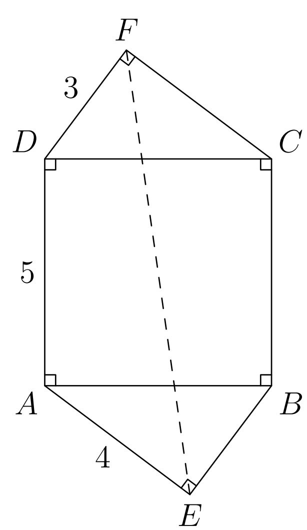
18. ________________ Let (a_n) be a sequence where for all $k > 2$,

$$a_k = a_{k-1} - a_{k-2} + 1.$$

If $a_1 = 5$ and $a_2 = 22$, what is $a_1 + a_{100}$?

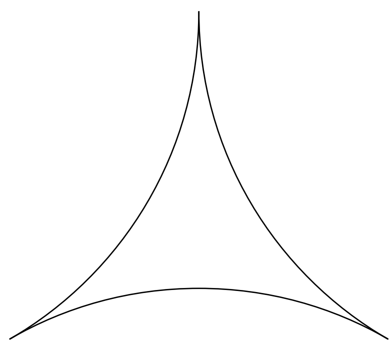
19. ________________ Alice, Bob, Carl, and David are watching a movie in a private theatre room with exactly 6 seats. Note that 2 of the seats are empty. At the intermission, they get up from their seats and buy some concessions. Alice is the first to return to the theatre room and randomly selects a seat among the 6 seats to sit in for the second half of the movie. Bob now enters and randomly selects a seat among the remaining 5. This process is continued for Carl and David, in that order. Let $\frac{m}{n}$ be the probability in simplest form that exactly 2 of them sit in their original seats. Find $m + n$.

20. ________________ In the diagram below, $ABCD$ is a square and $\triangle ABE$ and $\triangle CDF$ are right triangles. If $ABCD$ has a side length of 5, $AE = 4$ and $DF = 3$, what is $(EF)^2$?



21. ______________ A social media app is used by 75 students. In the app, students can make connections, and each connection is between exactly two students. If Student X is connected with Student Y, then Student Y is also connected with Student X. Given that each student has between 3 and 6 connections, over all the possibilities, what is the absolute value of the difference between the maximum and minimum number of unordered pairs of students that are in a connection?

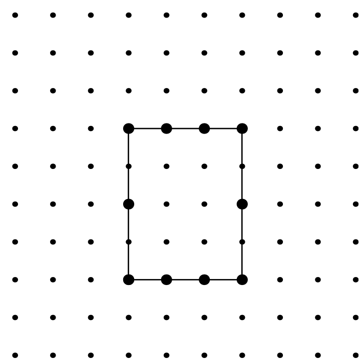
22. ______________ Let A be the enclosed area of the shape below, given that its perimeter is made of three 60° arcs, each of arc length 2π. If $A = m\sqrt{3} - n\pi$ where m and n are positive integers, find $m + n$.



23. ______________ Let $N = 20 \cdot 26 \cdot 2026$. Find the remainder when N is divided by 202.

24. ______________ Suppose n is an integer and $-2n^2 + 21n - 45$ is prime. What is $n^2 - n + 1$?

25. ______________ A farmer has 64 fenceposts that he would like to place on the unit grid. He later will connect these fenceposts with 4 straight fences that are parallel to the coordinate axes, forming a rectangular fenced enclosure. On the left and right fences, the farmer must space the fenceposts 2 meters apart. Meanwhile, on the top and bottom fences, the farmer must space the fenceposts 1 meter apart to protect the farm from wind gusts. Below is an example arrangement with 10 fenceposts. What is the maximum area, in square meters, that the farmer's fence can enclose?



26. ______________ The least common multiple function gives the smallest positive integer that is a multiple of all of its inputs. Let $x = \text{lcm}(26^{2026}, 2026^{26})$. Let y be the number of positive integer divisors of x. Let z be the number of positive integer divisors of y. What is z?

27. ______________ If $x + y = 1$ and $x^2 + y^2 = 2$, find $2 \cdot ((x + 1)^3 + (y + 1)^3)$. Note that x and y need not be rational numbers.

28. ________________ Anna cuts a regular octagon with side length 4 inches out of paper. She puts the tip of her pencil at the center of the octagon and spins it. As the octagon spins, the paths of its eight vertices trace out a circle. The area of the circle formed can be expressed in the form $(m+\sqrt{n})\pi$, where m and n are positive integers. Find $m+n$.

29. ________________ Abigail flips 6 fair coins in the following way.

1. Abigail flips all 6 coins, each one landing either heads up or tails up with equal probability.
2. If at least 3 of the coins are facing heads up, Abigail is done.
3. Otherwise, Abigail returns to step 1, starting the process again.

Abigail wants to know the probability that she ends up re-flipping the coins at least once and ends up with at least 5 coins facing heads up. If this probability can be expressed as $\frac{m}{n}$ where m and n are integers with no common factors, what is $m+n$?

30. ________________ Jane draws six points A, B, C, D, X and Y in the plane. Then, Jane draws a seventh point P at the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. Jane records the following observations:

- $PA \cdot PB \cdot PC \cdot PD = 324$.
- $2 \cdot PX = PY$.
- $\frac{PA}{PC} = \frac{PD}{PB}$.
- There is a circle that passes through the points A, B, X and Y.
- There is a circle that passes through the points C, D, X and Y.

What is the sum of all possible lengths of $\overline{XY}$?

31. ________________ **ESTIMATION.** How many digits does 200! have? Recall that $n!$ is the *factorial* of n, and it is equal to $n \times (n-1) \times (n-2) \cdots \times 2 \times 1$. For example, $4! = 4 \times 3 \times 2 \times 1 = 24$.

Remember that leaving this question blank means that you will be ranked the lowest among all students who tied with you on the math portion of the test.

PAGMT 2026, Senior Division, Solution Guidelines

Sayan Singh, Oscar Varodayan

1 Preface

For PAGMT, we provide solution guidelines rather than full solutions because we want students to experience the process of discovery for themselves. Mathematics is not just about reaching the correct answer, but about developing creative approaches, testing ideas, and learning how to reason independently. These guidelines are meant to offer direction when needed while still leaving room for students to explore, make connections, and build confidence in their own problem-solving abilities. We have also attached an answer key for students to compare their final answers with. We hope that everyone can gain insight and knowledge into creative problem solving and mathematics with the help of this document.

2 Solution Guidelines

1. Remember PEMDAS. Rewrite each division as multiplication by the reciprocal, then simplify carefully.

2. Read the question carefully. First apply the percentage discount to the original price. Then subtract the gift card amount.

3. Let the three integers be listed in increasing order. Use the average to find their total, then solve for the missing largest value.

4. List the sum of the row and column in each box, and then manually count the boxes that have a number inside them divisible by 3.

5. List possible triples by number of red, blue, and green pencils, making sure not to exceed the available quantity of each color.

6. Start with the total number of faces from both solids. Then subtract the two glued faces, since they are no longer visible.

7. Use the side ratio to write the sides as multiples of a common scale factor. Use the area formula to find the scale factor.

8. Work from the inside outward. Apply each function one step at a time, simplifying whenever possible. Make sure your calculations are correct!

9. Use the square's area to find its side length, then find its perimeter. Set that equal to the circle's circumference, and then solve for the radius.

10. Count the number of outcomes that give the required sum out of the total possible dice outcomes. Then simplify the fraction.

11. Factor the given number into powers of 2, 3, 5, and 7. Match the exponents to the variables. Then evaluate the requested expression.

12. Determine how many running intervals are needed to cover the full distance. The most common incorrect answer here was 88. Remember that Marvin does not need to rest after finishing. (So he runs, rests, runs, rests, ..., rests, runs. He does not rest at the end, as he is finished.)

13. Notice that factorials from a certain point onward all end in 0, meaning they don't affect the units digit of the sum. Only the first few factorials affect the units digit.

14. Rewrite the expression using powers of 2 and 5. Convert part of it into a power of 10, then count digits. (e.g. $61 * 10^5$ is 61 followed by 5 zeroes, so it has $2 + 5 = 7$ digits. Do the same for this value.)

15. Use angle facts about a regular pentagon and its diagonals. Identify all the isoceles triangles and mark their angles as equal. Derive a system of equations and solve for the desired angle.

16. The divisor-sum formula is the exact tool needed to solve this problem. Another way to solve it, albeit tedious, is to list the factors of 120 and manually add them up.

17. Model each state as an ordered pair showing the amount of water in each jug. Try systematic sequences of fill, empty, and transfer moves until one jug contains the target amount.

18. Compute the first several terms and look for a repeating pattern/cycle. Then use the cycle length to find the distant term.

19. Think of the second-half seating as a random assignment of four people to six seats. Count arrangements where exactly two people return to their original seats. Be patient and go through each case carefully.

20. There are many ways to solve this problem, most involving the Pythagorean Theorem and finding the heights of triangles. Try to make as many new right triangles as you can, and find all the lengths.

21. Use the handshaking idea: the total number of connections is half the sum of all students' degrees. Maximize and minimize the degree sum under the given restrictions. (The degree is the number of edges connected to that student.)

22. Use the fact that arc length equals radius times central angle. Find the radius of each 60° arc, then decompose the region into familiar geometric pieces by circumscribing one equilateral triangle with another of double the sidelength.

23. Simplify each factor modulo 202 before multiplying. Reduce after each multiplication to keep the numbers small and manageable.

24. Factor or rewrite the expression to see when it can be prime. Use the integer condition to narrow possible values of n.

25. Express the number of fenceposts needed in terms of the rectangle's width and height. Then maximize the area subject to the total post constraint. Make sure you don't make any off-by-one errors.

26. Prime factorize both large inputs. The LCM takes the larger exponent for each prime. Then use the divisor-counting formula twice to get to the answer.

27. Use $x + y$ and $x^2 + y^2$ to find xy. Then, think of how to get $x^3 + y^3$ from $x + y$, even if you get some extra terms. Then find how to cancel out those extra terms with xy and $x + y$.

28. The vertices trace a circle whose radius is the distance from the center of the octagon to a vertex. Use Pythagorean theorem to try to find twice the radius via a diagonal of the octagon.

29. First find the probability that one round causes a re-flip. Then find the probability that the final successful round has at least 5 heads. Combine these using the geometric nature of repeated trials. Also, Bayes' Theorem may be helpful here, given the conditional probability involved in this problem.

30. Use the two circle conditions involving A, B, X, Y and C, D, X, Y. Translate the products involving P into power-of-a-point relationships, then use the given ratio condition to determine possible values of XY. Note that there are two different configurations, and each configuration will give a different answer. Summing the two answers will give the final result.

31. **ESTIMATION.** Estimate the number of digits by approximating $\log_{10}(200!)$. Recall that $\log ab = \log a + \log b$. Use this property extensively to obtain the result.

3 Answer Key

1. 8
2. 32
3. 6
4. 9
5. 6
6. 22
7. 15
8. 138
9. 2
10. 41
11. 22
12. 85
13. 3
14. 24
15. 36
16. 360
17. 12
18. 2
19. 67
20. 98
21. 112
22. 54
23. 90
24. 13
25. 512
26. 12
27. 27
28. 144
29. 203
30. 12
31. 375