Palo Alto Girls Math Tournament 2025

Full name: _____

School: _____

Do not begin the test until you are instructed to do so. Read all rules carefully.

- This competition consists of 30 math problems and 1 estimation problem to be solved in 1 hour.
- Permitted materials: pencils, pens, erasers, scratch paper. Scratch paper will be provided by the proctors. Calculators, books, notes, rulers, compasses, protractors, or any other aids are prohibited.
- Answer format: Answers must be legibly written on the answer blank to be graded. Units are not required. For problems 1-30, all answers will be positive integers. For the estimation question, you may submit a decimal with a maximum of 3 digits after the decimal point.
- Diagrams are not necessarily to scale.
- Scoring: Your score will be the number of correct answers on questions 1-30. There is no penalty for guessing. Problem 31, the estimation question, will only be used for breaking ties. In the event of a tie, the student with a closer estimate to the actual answer will be ranked higher. If you leave the estimation question blank, you will be ranked last among all students who tied with you, so **it is highly recommended to answer the estimation question, even if you only spend a couple of seconds on it.**
- What if there is still a tie? In the unlikely case where two students tied and their estimations are equal, the person who solved the harder set of math questions will be ranked higher (the difficulty of the set of solved problems will be calculated by summing the problem numbers).
- Proctors may not answer any questions about the test problems.

Problem writers: Oscar Varodayan, Lucas Lin, Joycelyn Zhou, Nathan Ye, Jade Zhang, Beinan Ren, Krittika Chandra. Formatting & diagrams: Kaitlin Ye. Testsolvers: Ritwin Narra, Inhoo Chang, Raymond Zhou, Leo Varodayan.

1. _____ What is $2025 - 44^2$?

2. _____ Lucas is having a birthday party! He and his 10 friends each get a \$2.50 cupcake and a \$1.50 soda. They also buy a pack of candles for \$1.00. How much will this cost altogether?

3. _____ Carissa is 28 years older than her daughter Marissa. If Carissa is 33 years old, in how many years will she be 3 times as old as Marissa?

4. _____ How many positive perfect squares less than or equal to 100 have the same number of odd digits as even digits? Leading 0s should not be counted towards the number of even digits.

5. _____ Evaluate the square root of the square root of $2025 \cdot 25$.

6. _____ Kaitlin has three quadratics: $x^2 + 5x + 6$, $x^2 - 6x - 1$, and $2x^2 - 36x + 130$. She writes down the two roots of each quadratic. What is the sum of the six numbers she writes?

7. _____ Krittika and Oscar both get a weekly allowance, but Oscar gets \$5 more than Krittika each week. Each of them spends exactly \$20 a week eating lunch at Town and Country and saves the rest of their allowance. They both started with \$0, and after some number of weeks, Oscar has saved \$72, while Krittika only has saved \$42. How much allowance does Oscar receive each week?

8. _____ Beinan is currently writing problems for the super fun Palo Alto Girls Math Tournament. He takes 18 minutes to write an algebra problem and 12 minutes to write a combinatorics problem. Yesterday, Beinan wrote n problems, which took him exactly two hours. How many values of n are possible?

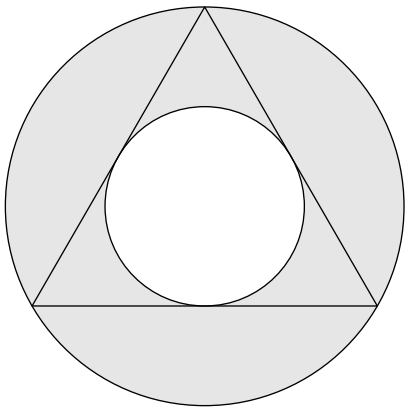
9. _____ At a high school called Pally, 90 students play a sport. There are 55 students on the track team, 34 on the swim team, and 29 on the volleyball team. 2 students play all three sports. How many students play exactly two sports?

10. _____ Mr. Mercado assigns the values 1, 2, 3, and 4 to the variables a, b, c, and d in a way such that no value is repeated, and $a^{b^{c^d}}$ is as big as possible. Find $1000a + 100b + 10c + d$.

11. _____ Five points labeled P, A, G, M, and T are on a line (not necessarily in that order). Given that $PA = 3$, $AG = 2$, $GM = 1$, $MT = 7$, and $TP = 9$, what is AT?

12. _____ Joycelyn and Jade each roll a standard 6-sided die. Then Nathan flips a fair coin. If it lands heads, Joycelyn and Jade add their two numbers, and if it lands tails, they multiply their two numbers. The probability that the result is a prime number can be expressed as $\frac{a}{b}$, where a and b are relatively prime integers. Find $a + b$.

13. _____ There is an equilateral triangle of side length 1, with one circle inscribed inside and one circle circumscribed outside. The area of the region inside the larger circle but outside the smaller circle can be expressed as π/n. Find n.



14. _____ Two high schools, Pally and Guun, each send a team to a math competition. There are three rounds, and only integer scores are possible. Pally scores 15 points higher than Guun on round 1, but Guun scores 25 points higher than Pally on round 2. At this point, Guun's total score is 1.05 times Pally's total score. However, on round 3, Pally makes a comeback, and gets a full score on the round, resulting in a total competition score of 250 points for Pally. How many points was round 3 worth?

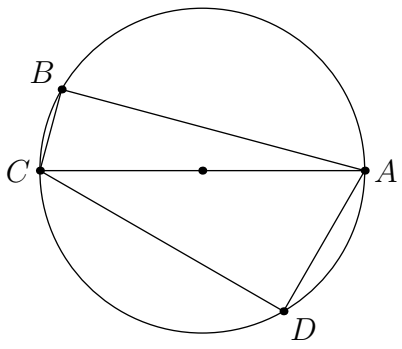
15. _____ How many positive divisors does $\frac{2026!}{2023!} + 2025$ have?

16. _____ Find the number of trailing zeros of the number:

$$10! + 11! + 12! + 13! + 14! + 15! + 16! + 17! + 18! + 19! + 20!$$

Recall that $n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$.

17. _____ Cyclic quadrilateral $ABCD$ has four distinct integer side lengths, and its circumcircle has diameter AC of length 25. What is the perimeter of $ABCD$?



18. _____ How many pairs of parallel edges are there in a regular hexagonal prism? Two lines are parallel if they lie in the same plane, but never intersect.

19. _____ What is the sum of all prime numbers that can be expressed as $n^3 + 5n - 3$ where n is a positive integer?

20. _____ A combinatorics class has four students, Amy, Bob, Calvin, and Debbie. For their midterm exam, they decide to form study groups to prepare. A study group may be as small as 1 student or as large as 4 students, and every student is in exactly one group. How many combinations of study groups are possible?

21. _____ Mr. Orz is making cookies. He has a rectangular sheet of batter that is 18 inches long and 22 inches wide. He wants to cut out three identical circles in this sheet of batter such that no two circles overlap. What is the maximum possible radius of the circles?

22. _____ How many (not necessarily proper) subsets of $\{1, 2, 3, 4, 5, 6\}$ have a pair of two elements which are consecutive integers?

23. _____ Consider the function:

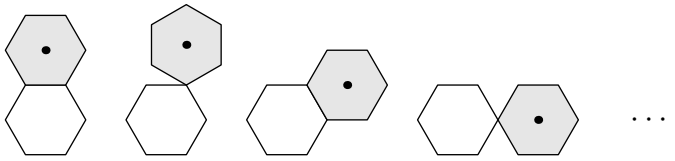
$$f(x) = \begin{cases} \frac{x}{2}, & x \text{ is even,} \\ 2x + 4, & x \text{ is odd.} \end{cases}$$

How many positive integers m exist such that $1 < m < 100$ and $f(f(f(\cdots f(m)))) = 1$ is satisfied for some number of iterations of f ?

24. _____ Emily is dancing with her friend Amily at the Pally Swing Dance. The two friends are always 2 feet apart. Emily rotates about Amily in a $45°$ arc counterclockwise. Then, Amily rotates around Emily in a $90°$ arc clockwise. Emily rotates around Amily in a $135°$ arc counterclockwise. Then Amily rotates around Emily in an $180°$ arc clockwise. The distance between Amily's starting and ending points can be expressed as $a + b\sqrt{c}$, where a, b, and c are integers and c has no perfect square factors other than 1. Find $a + b + c$.

25. _____ The integers 1 through 6 are randomly ordered. Marco will be *happy* if for each of the 5 pairs of adjacent numbers, the 2 numbers are relatively prime. The probability Marco is happy can be expressed as $\frac{a}{b}$, where a and b are relatively prime. Find $a + b$.

26. _____ There are two regular hexagons with side length 1, one shaded and one white. The two hexagons are placed so that they share exactly one side. Then, the shaded hexagon pivots about a vertex that they share such that it "rolls" to the next edge. It continues in this fashion until it returns to its original position. The area of the shape that is traced out by the path of the center of the shaded hexagon can be expressed as $a\pi + b\sqrt{c}$, where a, b, and c are integers and c has no perfect square factors other than 1. Find $a + b + c$.



27. _____ $20^5 + 25^5 + 202 + 5 + 2 + 0^{25}$ is an 8-digit number where each digit is a distinct digit between 1 and 9. Which digit between 1 and 9 does not show up?

28. _____ How many triples (a, b, c) of nonnegative integers exist such that if Bessy has a 25¢ coins, b \$1 bills, and c \$5 bills, she will have exactly \$20.25?

29. _____ In $\triangle ABC$, $AB = 4$, $AC = 5$, and $BC = 6$. What is the square of the length of the angle bisector of $\angle ABC$?

30. _____ Simplify:

$$\frac{1}{\log_{10} 2} \cdot \left(\frac{\log_{10}(2^3)}{\log_{10}(3^2)} \cdot \frac{\log_{10}(3^4)}{\log_{10}(4^3)} \cdot \frac{\log_{10}(4^5)}{\log_{10}(5^4)} \cdots \frac{\log_{10}(99^{100})}{\log_{10}(100^{99})} \right).$$

31. _____ **ESTIMATION.** What will be the average score on the math portion of this test (problems 1-30) across all competitors?

Remember that your guess for this problem does not have to be an integer. You can specify up to 3 digits after the decimal point. Leaving this question blank means that you will be ranked the lowest among all students who tied with you on the math portion of the test.

Answer key:

1. 89
2. 45
3. 9
4. 5
5. 15
6. 19
7. 32
8. 4
9. 24
10. 2341
11. 6
12. 31
13. 4
14. 50
15. 91
16. 2
17. 66
18. 33
19. 3
20. 15
21. 5
22. 43
23. 6
24. 8
25. 11
26. 8
27. 7
28. 55
29. 18
30. 25
31. 9.126760563